General method according to EN 1993-1-1, §6.3.4

Formulation. Methodology and applications in the context of CAE

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Content of presentation

- What is the General Method?
- When is it used?
- Formulation
- Some examples
- Limitations
- Conclusion
What is the General Method?

- Evaluation of member/2D frame stability: in-plane & out-of-plane
- Simple formulation
- Difficulty: in deriving the 2 essential parameters ($\alpha_{ult,k}$ & $\alpha_{cr,op}$)
- Requires the use of FE software
- May apply to:
  - complex structural components
  - structural components with complex restraint conditions & load
Applications of the General Method

**Why:** resistance for lateral (FB) and lateral-torsional buckling (LTB)

**When:** where §6.3.1 -- §6.3.3 do not apply

**For:**
- single members (*built-up* or not, *uniform* or not, with *irregular support* conditions or not)
- plane *frames* or *sub-frames* composed of such members

**Load:** compression and/or (mono-axial) *in-plane* bending and shear

**To watch out:** no plastic hinge rotates!

**The National Annex may specify** the field and limits of application of this method
Applications of the General Method

- **NBN EN 1993-1-1 ANB:**
  - Accepted as a general procedure
  - Only when EC3-1-1, §6.3.1 -- §6.3.3 CANNOT be applied

- **NF EN 1993-1-1 NA:**
  - Requires that in-plane stability be determined from elastic analysis of the whole structure

- **DIN EN 1993-1-1 NA:**
  - Limits to I-sections only
  - In-plane stability is limited by formation of first plastic hinge

- **BS EN 1993-1-1 NA:**
  - Nominally straight components
Applications of the General Method

  - variable sections, haunches
  - “odd” lateral restraints, different from “simple fork”
  - complex moment distribution

=> Common conditions in portal frames

Examples of structural components in portal frames that can be treated using EN1993-1-1 §6.3.4
Intermezzo: Stability design of steel frames

- Design against lateral or lateral-torsional buckling:
  - Imperfections:
    - **Global imperfections** of the frame
    - Local (member) imperfections
  - Deformations:
    - **Second-order** deformation in the frame (nodal displacements)
    - Local (member) **second-order** deformation

- For uniform members: contained in EN 1993-1-1, Chapters §6.3.1 to §6.3.3
Intermezzo: Stability design of steel frames

Analytical methods according to EN 1993-1-1, 6.2 & 6.3.1 to 6.3.3

Interaction

Compression

Section check...
- 6.2.4 Compression

Stability check...
- 6.3.1 Uniform members in compression (Flexural buckling)

Interaction

- 6.2.8 Bending and shear
- 6.2.9 Bending and axial force

Bending in-plane

Section check...
- 6.2.5 Bending moment
- 6.2.6 Shear

Stability check...
- 6.3.2 Uniform members in bending (Lateral-torsional buckling)

Interaction

- 6.3.3 Uniform members in bending and axial compression

Annex A/Annex B
Intermezzo: Stability design of steel frames

Section checks & general method according to EN 1993-1-1, 6.2 & 6.3.4

- Compression
  - Section check…
  - Stability check…
  - 6.2.4 Compression
- Interaction
  - 6.2.8 Bending and shear
  - 6.2.9 Bending and axial force
- Bending in-plane
  - Section check…
  - Stability check…
  - 6.2.5 Bending moment
  - 6.2.6 Shear
- General method
  - 6.3.4 General method for lateral and lateral torsional buckling of structural components
Formulation of the General Method

- Analysis of the whole structure with global imperfections

In-plane stability (FB around y-y)

G(M)NIA
- Fully restrained out-of-plane

Out-of-plane stability (FB around z-z, LTB)

LBA
- Account for warping

Selected member/2D frame

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]

\[ \chi_{op} = \min(\chi, \chi_{LT}) \]

Buckling curves for critical section

Stability verification based on General Method

\[ \chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1 \]
Formulation of the General Method

Analysis of the whole structure with global imperfections

Option 1

Load: internal forces from global model & local load
BCs: hinges at extremities

Option 2

Load: internal forces from global model & local load
BCs: hinges at extremities

global analysis model
Formulation of the General Method

In-plane stability (FB around y-y)

Out-of-plane stability (FB around z-z, LTB)

G(M)NIA
Fully restrained out-of-plane

LBA
Account for warping

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]

Selected member/2D frame

Analysis of the whole structure with global imperfections

Buckling curves for critical section

\[ \chi_{op} = \min(\chi, \chi_{LT}) \]

Stability verification based on General Method

\[ \chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1 \]
Formulation of the General Method

Analysis of the whole structure with global imperfections

Selected member/2D frame

In-plane stability (FB around y-y)

G(M)NIA

Fully restrained out-of-plane

LBA

Account for warping

Out-of-plane stability (FB around z-z, LTB)

How much we have to increase the design load in order to:

- Reach the characteristic resistance of the most critical cross-section,…
- … considering the member/frame in-plane behaviour: in-plane geometrical deformation and imperfections…
- … without taking lateral or lateral torsional buckling into account

$$\lambda_{op} = \min(\chi, \chi_{LT})$$

$$\alpha_{ult,k}$$
Formulation of the General Method & Example

2D frame

- members of **variable height**
- “design load:” **10 kN/m**
- **slender** sections
Formulation of the General Method & Example

Model: 2\textsuperscript{nd} order analysis of the \textit{frame} taking into account \textbf{all in-plane effects}:

- \textbf{In-plane stability} (FB around y-y)
  
  \textbf{G(M)NIA}  
  Fully restrained out-of-plane

- $\alpha_{\text{ult,k}}$

- 3 DoF \textbf{beam} elements
- \textbf{~10 sections} per member
- per section: (effective) \textbf{cross-section properties} and \textbf{class}
- global and member \textbf{imperfections}
Formulation of the General Method & Example

Model: 2\textsuperscript{nd} order analysis of the frame taking into account all in-plane effects:

- In-plane stability (FB around y-y)
- G(M)NIA
  - Fully restrained out-of-plane

\[ \alpha_{ult,k} = 2.16 \geq 1 \]

- Run analysis
  - Make sure no plastic hinge rotates!
- Derive unity Cross-Section Check value per section
- \(\Rightarrow\) Maximal unity check == critical section
Formulation of the General Method

In-plane stability (FB around y-y)

G(M)NIA
Fully restrained out-of-plane

LBA
Account for warping

Out-of-plane stability (FB around z-z, LTB)

\[ \chi_{op} = \min(\chi, \chi_{LT}) \]

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]

\[ \chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1 \]

Analysis of the whole structure with global imperfections

Selected member/2D frame

Buckling curves for critical section

Stability verification based on General Method
Formulation of the General Method

- Analysis of the whole structure with global imperfections

- In-plane stability (FB around y-y)
  - G(M)NIA: Fully restrained out-of-plane
- Out-of-plane stability (FB around z-z, LTB)
  - LBA: Account for warping

How much we have to increase the design load in order to:

- Reach the elastic critical resistance of the structural component, ...:
  \[ \alpha_{cr,op} \]
  
- ... with regards to lateral or lateral-torsional buckling...
  \[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]
  
- ... without taking in-plane flexural buckling into account
  \[ \chi_{op} = \min(\chi, \chi_{LT}) \]

\[ \chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1 \]
Formulation of the General Method

**Linear stability** analysis of the frame **ignoring all in-plane effects**:

- Elastic analysis
- No imperfections needed
- Warping deformation needed:
  - Shell elements
  - 7DoF beam elements
- The first eigen mode representing LB or LTB is the critical
- $\alpha_{cr,op}$: the load factor for this critical mode
- $\alpha_{cr,op} \geq 1$
Formulation of the General Method & Example

Linear stability analysis of the frame ignoring all in-plane effects:

- Fork support
- Lateral restraint
- Shell elements
- Mesh size 5 cm
- LBA
  Account for warping
- Out-of-plane stability (FB around z-z, LTB)
- $\alpha_{cr,op}$
Formulation of the General Method & Example

Linear stability analysis of the frame ignoring all in-plane effects:

Critical mode shape: No 1
Load factor: 2.12

LBA
Account for warping

Out-of-plane stability (FB around z-z, LTB)

\[ \alpha_{cr,op} = 2.12 \geq 1 \]
Formulation of the General Method

In-plane stability (FB around y-y)

G(M)NIA
Fully restrained out-of-plane

Out-of-plane stability (FB around z-z, LTB)

LBA
Account for warping

Analysis of the whole structure with global imperfections

Selected member/2D frame

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]

\[ \chi_{op} = \min(\chi, \chi_{LT}) \]

\[ \chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1 \]

Buckling curves for critical section

Stability verification based on General Method
Formulation of the General Method

Analysis of the whole structure with global imperfections

In-plane stability (FB around y-y)

G(M)NIA
Fully restrained out-of-plane

Out-of-plane stability (FB around z-z, LTB)

LBA
Account for warping

\[ \chi_{op} = \min(\chi, \chi_{LT}) \]

Relative slenderness of member/(sub)frame

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]

Buckling curves for critical section

Reduction factor of the resistance; takes into account all out-of-plane effects

\[ \chi_{cr,op}, \chi_{LT} \geq 1 \]
Formulation of the General Method

\[
\bar{\lambda}_{op} = \frac{\sqrt{\alpha_{ult,k}/\alpha_{cr,op}}}{\alpha_{ult,k}/\alpha_{cr,op}}
\]

Critical section from calculation of \( \alpha_{ult,k} \)

EN 1993-1-1, §6.3.1

\[
\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}
\]

\[
\Phi = 0.5 \left[ 1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2 \right]
\]

\( \chi = 1 \)

EN 1993-1-1, §6.3.2

\[
\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}}
\]

\[
\Phi_{LT} = 0.5 \left[ 1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right]
\]

\( \chi_{LT} = 1 \)

\( \bar{\lambda}_{op} > 0.2 \)

FB \rightarrow LTB

\( \bar{\lambda}_{op} > \bar{\lambda}_{LT,0} \)

YES

\( \chi_{op} = \min(\chi, \chi_{LT}) \)

NO

YES

Table 6.1: Imperfection factors for buckling curves

<table>
<thead>
<tr>
<th>Buckling curve</th>
<th>( a_0 )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperfection factor ( \alpha )</td>
<td>0.13</td>
<td>0.21</td>
<td>0.34</td>
<td>0.49</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 6.3: Recommended values for imperfection factors for lateral torsional buckling curves

<table>
<thead>
<tr>
<th>Buckling curve</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperfection factor ( \alpha_{LT} )</td>
<td>0.21</td>
<td>0.34</td>
<td>0.49</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Formulation of the General Method & Example

- \( \alpha_{ult,k} = 2.16 \geq 1 \)
- \( \alpha_{cr,op} = 2.12 \geq 1 \)

\[ \bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = \sqrt{\frac{2.16}{2.12}} = 1.01 \]

- At critical section: FB curve \( b \), \( \alpha = 0.34 \)

\[ \phi = 0.5 \left[ 1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2 \right] = 1.15 \]

\[ \chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} = 0.59 \]

At critical section: LTB curve \( d \), \( \alpha_{LT} = 0.76 \)

\[ \phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 1.32 \]

\[ \chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} = 0.46 \]

\[ \chi_{op} = \min(\chi, \chi_{LT}) = 0.46 \]
Formulation of the General Method

Analysis of the whole structure with global imperfections

Selected member/2D frame

In-plane stability (FB around y-y)

G(M)NIA
Fully restrained out-of-plane

LBA
Account for warping

Out-of-plane stability (FB around z-z, LTB)

$\lambda_{op} = \frac{\alpha_{ult,k}}{\alpha_{cr,op}}$

$\chi_{op} = \min(\chi, \chi_{LT})$

$\chi_{op} \alpha_{ult,k} / \gamma_{M1} \geq 1$

Buckling curves for critical section

Stability verification based on General Method
Formulation of the General Method

Analysis of the whole structure with global imperfections

Selected member/2D frame:

- In-plane stability (FB around y-y)
- G(M)NIA: Fully restrained out-of-plane
- LBA: Account for warping
- Out-of-plane stability (FB around z-z, LTB)

In-plane

Out-of-plane

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \]

\[ \chi_{op} = 0.46 \]

Buckling curves for critical section

\[ \alpha_{ult,k} = 2.16 \]

Stability verification based on General Method

\[ 0.46 \times 2.16 \div 1.0 = 0.994 \approx 1 \]
Example 2

From: “Background document to EN 1993-1-1,”
G. Sedlacek, J. Naumes, 2009

- A support frame from the Schwebebahn in Wuppertal
- Variable cross-section
- Fork supports modelled at column feet
- Beam laterally supported eccentrically in 2 points
- Asymmetric loading
- Non-uniform distribution of N and M_y
Example 2

From: “Background document to EN 1993-1-1,”
G. Sedlacek, J. Naumes, 2009

With FEM, see Figure 4.7 the numerical values are

\[ \alpha_{\text{ult},k,min} = 1.69 \]
\[ \alpha_{\text{crit}} = 3.41 \]
For the verification flexural buckling curve $c$ has been used as safe-sided approach. All relevant calculation steps are given in Figure 4.6.

Figure 4.6: Example for the lateral torsional buckling verification acc. to the general method
Limitations of the General Method

  - Linear members and truss and frame structures built-up out of linear members, where the lateral (torsional) buckling is related to a straight member behaviour. Sufficient lateral supports should be present such that the behaviour with respect to overall buckling between these lateral supports can be regarded as a straight member behaviour.
  - For structures/components out of the scope of the general method, a rigorous method like 3D GMNIA should be applied.
  - The General Method is always on the safe side when compared to a full 3D GMNIA.
Conclusion

- The general method is **used for stability** verifications for:
  - standard or complex structural components
  - with complex loading or boundary conditions (or not)
  - loaded in compression and/or in-plane bending

- The method takes into account:
  - **in-plane** imperfections & loss of in-plane stability by GMNIA of the structural component
  - **out-of-plane** loss of stability by LBA of the component (eventually, by the reduction factor $\chi_{op}$)
  - **out-of-plane** imperfections by adopting the appropriate buckling curve in the derivation of $\chi_{op}$
References


### General method in context

<table>
<thead>
<tr>
<th>Internal forces from...</th>
<th>Modelled imperfections</th>
<th>EN 1993-based checks required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order analysis</td>
<td>none</td>
<td>§6.2 &amp; in-plane FB $L_b \neq L$ out-of-plane FB /LTB (§6.3)</td>
</tr>
<tr>
<td>2D (in-plane) 2nd order analysis</td>
<td>In-plane global inclination</td>
<td>§6.2 &amp; in-plane FB $L_b = L$ out-of-plane FB /LTB (§6.3)</td>
</tr>
<tr>
<td>2D (in-plane) 2nd order analysis</td>
<td>In-plane (1) global inclination &amp; (2) member imperfections</td>
<td>§6.2 &amp; out-of-plane FB /LTB (§6.3)</td>
</tr>
<tr>
<td>3D 2nd order analysis, warping included</td>
<td>Global inclination Member imperfections</td>
<td>Section checks only (§6.2)</td>
</tr>
<tr>
<td>(General method) 1st order for whole structure 2nd order for component/frame</td>
<td>In-plane (1) global inclination &amp; (2) member imperfections</td>
<td>$\chi_{op} a_{ult,k} / \gamma_{M1} \geq 1$</td>
</tr>
</tbody>
</table>
Thank you for your attention

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